## Key to Final Exam S1 Computer Architecture

Duration: 1 hr. 30 min.

Last name: $\qquad$ First name: $\qquad$ Group: $\qquad$

## Write answers only on the worksheet. Do not show any calculation unless you are explicitly asked. Do not use a pencil or red ink.

## Exercise 1 (2 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not $1 / 4$ or $2^{-2}$ ).

| Number to Convert | Source Form | Destination Form | Result |
| :---: | :---: | :---: | :---: |
| 101011011.01011 | Binary | Decimal | $\mathbf{3 4 7 . 3 4 3 7 5}$ |
| B09.58 | Hexadecimal | Decimal | $\mathbf{2 8 2 5 . 3 4 3 7 5}$ |
| 999 | Decimal | Base 9 | $\mathbf{1 3 3 0}$ |
| 3245.43 | Base 8 | Hexadécimale | $\mathbf{6 A 5 . 8 C}$ |

## Exercise 2 (5 points)

Perform the following 8 -bit binary operations (the two operands and the result are 8 bits wide). Then, convert the result into unsigned and signed decimal values. If an overflow occurs, write down 'ERROR' instead of the decimal value.

| Operation | Binary Result | Decimal Value |  |
| :---: | :---: | :---: | :---: |
|  |  | Signed |  |
| $11000111+10000101$ | 01001100 | ERROR | ERROR |
| $01010110-11110101$ | 01100001 | ERROR | $\mathbf{9 7}$ |
| $00101110-10101100$ | 10000010 | ERROR | ERROR |
| $11010001+00001010$ | 11011011 | 219 | $\mathbf{- 3 7}$ |
| $01101011-01001000$ | 00100011 | $\mathbf{3 5}$ | $\mathbf{3 5}$ |

## Exercise 3 (5 points)

According to the truth table, complete the the Karnaugh maps below (draw also the circles). Then, give the most simplified expression for $W, X, Y$ and $Z$ (do not simplify by using the EXCLUSIVE-OR operator). No points will be given to an expression if its Karnaugh map is wrong. Note that when $D C B A>1001_{2}$, then $W, X, Y$ and $Z$ are undefined.

| $\mathbf{D}$ | $\mathbf{C}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

DC | $\mathbf{W}$ | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{0 1}$ | 0 | 1 | 1 | 1 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 1 | 1 | $\Phi$ | $\Phi$ |

$\mathbf{w}=\mathrm{D}+\mathrm{C} . \mathrm{A}+\mathrm{C} . \mathrm{B}$
BA

DC | $\mathbf{Y}$ | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 0 | 1 |
| $\mathbf{0 1}$ | 0 | 0 | 1 | 0 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 0 | 0 | $\Phi$ | $\Phi$ |

$\mathbf{Y}=\mathrm{C} \cdot \mathrm{B} \cdot \mathrm{A}+\overline{\mathrm{C}} \cdot \mathrm{B} \cdot \overline{\mathrm{A}}$

| $\mathbf{B A}$ |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{X}$ $\mathbf{0 0}$ $\mathbf{0 1}$ $\mathbf{1 1}$ $\mathbf{1 0}$ <br> $\mathbf{0 0}$ 0 0 1 0 <br> $\mathbf{0 1}$ 1 0 0 0 <br> $\mathbf{1 1}$ $\Phi$ $\Phi$ $\Phi$ $\Phi$ <br> $\mathbf{1 0}$ 1 1 $\Phi$ $\Phi$ |  |  |  |  |  |  |

$\mathbf{X}=\mathrm{D}+\mathrm{C} \cdot \overline{\mathrm{B}} \cdot \overline{\mathrm{A}}+\overline{\mathrm{C}} \cdot \mathrm{B} \cdot \mathrm{A}$
BA

DC | $\mathbf{Z}$ | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 1 | 0 | 0 |
| $\mathbf{0 1}$ | 1 | 0 | 0 | 1 |
| $\mathbf{1 1}$ | $\Phi$ | $\Phi$ | $\Phi$ | $\Phi$ |
| $\mathbf{1 0}$ | 0 | 1 | $\Phi$ | $\Phi$ |

$\mathbf{Z}=\mathrm{C} . \overline{\mathrm{A}}+\overline{\mathrm{C}} \cdot \overline{\mathrm{B}} . \mathrm{A}$

Finally, simplify $Y$ by using the EXCLUSIVE-OR operator:

$$
\mathbf{Y}=\text { C.B.A }+\overline{\mathrm{C}} \cdot \mathrm{~B} \cdot \overline{\mathrm{~A}}=\mathrm{B} \cdot(\mathrm{C} \cdot \mathrm{~A}+\overline{\mathrm{C}} \cdot \overline{\mathrm{~A}})=\mathbf{B} \cdot \overline{\mathbf{C} \oplus \mathbf{A}}
$$

## Exercise 4 (3 points)

We want to simplify the following circuit diagram:


1. Without any simplifications, give the $S$ output in terms of $a, b$ and $c$.

$$
\mathrm{S}=\overline{\mathbf{a} . \mathrm{b}+\mathrm{a} . \mathrm{c}}+\overline{\mathbf{a}} \overline{\mathrm{b}} . \bar{c}
$$

2. Give the most simplified expression of $S$.

$$
\mathbf{S}=\overline{\mathbf{a}}+\overline{\mathbf{b}} . \overline{\mathbf{c}}
$$

3. From the most simplified expression, draw a new circuit diagram by using three NOT gates, one twoinput AND gate and one two-input OR gate.


## Exercise 5 (5 points)

Let us consider the three following expressions:
$\mathrm{S} 1=\overline{(\mathrm{A}+\overline{\mathrm{B}}+\mathrm{C}) \cdot(\mathrm{A}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}})}$
$\mathrm{S} 2=\mathrm{A} \cdot \mathrm{B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \mathrm{B} \cdot \overline{\mathrm{C}}+\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \mathrm{C}$
$\mathrm{S} 3=\mathrm{A} \oplus(\mathrm{B} . \overline{\mathrm{C}})$

1. Give the most simplified expression of $S 1$. The result must be given as a sum of products (without parentheses).

S1 $=\mathbf{B}+\mathbf{A} . \mathbf{C}$
2. Write down the minterm canonical form of S1.
$\mathbf{S 1}=\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C}+\mathbf{A} \cdot \mathbf{B} \cdot \overline{\mathrm{C}}+\overline{\mathbf{A}} \cdot \mathrm{B} \cdot \mathrm{C}+\overline{\mathbf{A}} \cdot \mathbf{B} \cdot \overline{\mathrm{C}}+\overline{\mathbf{A}} \cdot \overline{\bar{B}} \cdot \mathbf{C}$
3. Write down the maxterm canonical form of S2.
$\mathbf{S} 2=(\mathbf{A}+\mathbf{B}+\mathbf{C}) \cdot(\mathbf{A}+\mathbf{B}+\overline{\mathbf{C}}) \cdot(\mathbf{A}+\overline{\mathrm{B}}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\mathbf{C})$
4. Is it true that S2 = S3? (Answer "Yes" or "No")

## Yes

5. Determine the $m$ and $n$ integers so that the following identity is true: $2^{m}-2^{n}=4064$
$\mathrm{m}=12$

$$
n=5
$$

Feel free to use the blank space below if you need to:

